$\boxed{35}$ $y=x+2$, $y=-2x+1$ と x軸正の向きとのなす角をそれぞれ θ_1, θ_2 とすると

$$\tan\theta_1 = 1 \quad , \quad \tan\theta_2 = -2$$

よって

$$\tan\theta = \tan(\theta_2 - \theta_1)$$

$$= \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1}$$

$$= \frac{-2-1}{1+(-2)\cdot 1}$$

$$= \frac{-3}{1-2} = 3$$

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(1) $\sin 3\alpha = \sin(2\alpha + \alpha)$

$= \sin 2\alpha \cos\alpha + \cos 2\alpha \sin\alpha$

$= 2\sin\alpha\cos^2\alpha + (1 - 2\sin^2\alpha)\cdot\sin\alpha$

$= 2\sin\alpha(1 - \sin^2\alpha)$

$+ \sin\alpha - 2\sin^3\alpha$

$= 3\sin\alpha - 4\sin^3\alpha$

$\cos 3\alpha = \cos(2\alpha + \alpha)$

$= \cos 2\alpha \cos\alpha - \sin 2\alpha \sin\alpha$

$= (2\cos^2\alpha - 1)\cos\alpha$

$- 2\sin^2\alpha\cos\alpha$

$= 2\cos^3\alpha - \cos\alpha$

$- 2(1 - \cos^2\alpha)\cos\alpha$

$= 2\cos^3\alpha - \cos\alpha - 2\cos\alpha$

$+ 2\cos^3\alpha$

$= 4\cos^3\alpha - 3\cos\alpha$

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(2)

$\theta = 18^\circ$ とおくと　$5\theta = 90^\circ$

ここで　$\sin 2\theta = \sin(5\theta - 3\theta)$

$= \sin(90^\circ - 3\theta)$

$= \cos 3\theta$

よって

$$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$$

$\cos\theta = \cos 18^\circ \neq 0$ なので 両辺を

$\cos\theta$ でわって

$$2\sin\theta = 4\cos^2\theta - 3$$

$$= 4(1 - \sin^2\theta) - 3$$

$$= 4 - 4\sin^2\theta - 3$$

$$= 1 - 4\sin^2\theta$$

$$\therefore\ 4\sin^2\theta + 2\sin\theta - 1 = 0$$

$$\sin\theta = \frac{-1 \pm \sqrt{1+4}}{4} = \frac{-1 \pm \sqrt{5}}{4}$$

$0 < \sin\theta < 1$ なので

$$\sin\theta = \sin 18^\circ = \frac{-1+\sqrt{5}}{4}$$

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$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

$$\tan x = \frac{2t}{1-t^2}$$

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(1) $y = (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$

$= \cos^2 x - \sin^2 x$

$= \cos 2x$

よって 最大値は 1

($x = n\pi$ のとき)

最小値は -1

($x = n\pi + \frac{\pi}{2}$ のとき)

ただし n は整数.

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(2) $y = (\cos^2 x + \sin^2 x)^2$

$- 2\cos^2 x \sin^2 x$

$= 1 - \frac{1}{2}\sin^2 2x$

よって最大値は 1

$\left(x = \frac{n\pi}{2} \text{ のとき} \right)$

最小値は $\frac{1}{2}$

$\left(x = \pm\frac{\pi}{4} + n\pi \text{ のとき} \right)$

ただし n は整数

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(1) $\sin\left(x+\frac{\pi}{2}\right) = \sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2}$

$= \sin x \cdot 0 + \cos x \cdot 1$

$= \cos x$

$\sin(x+\pi)$

$= \sin x \cos\pi + \cos x \cdot \sin\pi$

$= \sin x \cdot (-1) + \cos x \cdot 0$

$= -\sin x$

$\sin\left(x+\frac{3}{2}\pi\right)$

$= \sin x \cdot \cos\frac{3}{2}\pi + \cos x \cdot \sin\frac{3}{2}\pi$

$= \sin x \cdot 0 + \cos x \cdot (-1)$

$= -\cos x.$

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(2) $\cos\left(x+\frac{\pi}{2}\right) = \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}$

$= \cos x \cdot 0 - \sin x \cdot 1 = -\sin x.$

$\cos(x+\pi)$

$= \cos x \cos\pi - \sin x \sin\pi$

$= \cos x \cdot (-1) - \sin x \cdot 0$

$= -\cos x$

$\cos\left(x+\frac{3}{2}\pi\right)$

$= \cos x \cdot \cos\frac{3}{2}\pi - \sin x \cdot \sin\frac{3}{2}\pi$

$= \cos x \cdot 0 - \sin x \cdot (-1)$

$= \sin x.$

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(3) m を正の整数とする

㋐ $n = 4m$ のとき

$$\sin\left(x + \frac{n\pi}{2}\right) = \sin x$$

$$\cos\left(x + \frac{n\pi}{2}\right) = \cos x$$

㋑ $n = 4m + 1$ のとき

$$\sin\left(x + \frac{n\pi}{2}\right) = \cos x$$

$$\cos\left(x + \frac{n\pi}{2}\right) = -\sin x$$

㋒ $n = 4m + 2$ のとき

$$\sin\left(x + \frac{n\pi}{2}\right) = -\sin x$$

$$\cos\left(x + \frac{n\pi}{2}\right) = -\cos x$$

㋓ $n = 4m + 3$ のとき

$$\sin\left(x + \frac{n\pi}{2}\right) = -\cos x$$

$$\cos\left(x + \frac{n\pi}{2}\right) = \sin x$$